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High-Angle Formulation for the Nonlinear Progressive-wave Equation (NPE) Model

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The Nonlinear Progressive-wave Equation (NPE) model has been reformulated for wider propagation angle accuracy. The wide-angle NPE results from range-integrating the second-order time domain nonlinear wave equation rather than reducing it to a one-way equation as in the original NPE. The numerical implementation of the second-order NPE retains two time levels rather than one. The resulting model is referred to as NPE2. Benchmark tests reveal that root mean square (rms) errors in the model are reduced by a factor of 10 for moderately high propagation angles as compared to similar tests of the first-order NPE.

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HIGH-ANGLE FORMULATION FOR THE NONLINEAR PROGRESSIVE-WAVE EQUATION (NPE) MODEL

INTRODUCTION

The Nonlinear Progressive-wave Equation (NPE) model [1,2] is part of a numerical simulation chain applied to hydroacoustic nonproliferation research. Simulations of ocean detonations [3] are performed by Lawrence Livermore National Laboratory using the strong shock CALE model (a hydrodynamic code). The NPE model takes input from CALE when shock strength is finite but too low for hydrocodes to perform accurately and efficiently. The NPE takes the weak shock input and propagates the wave until it can be joined [2] to linear ocean acoustic modes for propagation across entire ocean basins.

THEORY

The NPE model is based on the following time domain nonlinear wave equation emerging directly from the Euler equations of hydrodynamics:

$$\frac{\partial^2 \rho}{\partial t^2} = \nabla^2 p + \partial_i \partial_j (\rho \mathrm{v}_i \mathrm{v}_j), \tag{1}$$

using standard Einstein summation convention, where repeated indices are summed, and where ρ is density, p is pressure, and v_i is the fluid velocity. Equation (1) is recast in a wave-tracking frame moving horizontally in the r-direction at a speed c_0, descriptive of the sound speed of the propagation medium (the ocean in the present case). The time derivative D/Dt in the moving frame is $\partial/\partial t + c_0\, \partial/\partial r$. Equation (1) cast in the moving frame gives a left-hand side (lhs) that involves derivatives D_t^2, $c_0 \partial_r D_t$, and $c_0^2\, \partial_r^2$, where D_t is shorthand for D/Dt. For a progressive wave, the $c_0^2\, \partial_r^2$ term offsets a linear term in the Laplacian on the right-hand side (rhs) of Eq. (1) after invoking a nonlinear equation of state. The original NPE neglects D_t^2 in comparison with $c_0 \partial_r D_t$, and r-integrates the result to give

$$\frac{DR}{Dt} \approx -\frac{\partial}{\partial r}\left(c_1 R + c_0 \frac{\beta}{2} R^2 \right) - \frac{c_0}{2} - \frac{R}{r} - \frac{c_0}{2} \int_{r_f}^{r} \frac{\partial^2}{\partial z^2} dr, \tag{2}$$

where R is a dimensionless overdensity ρ'/ρ_0, c_1 is $c(\mathbf{r}) - c_0$, β is a dimensionless nonlinearity parameter derived from the equation of state (for water, $\beta \approx 3.5$), and r_f is an arbitrary point in the quiescent medium ahead of the wave. The neglect of the D_t^2 term in an azimuthally symmetric ocean is justified when the propagating wave is nearly cylindrical, but cannot be justified at early times when the wave is nearly spherical.

The high-angle NPE2 model is derived by retaining the D_t^2 term previously mentioned. We isolate the portion of Eq. (2) having to do with diffraction, give a derivation of the high-angle diffraction step, then reassemble the model by using the fractional step method. Namely, the corrected diffraction step is followed by a step in which the refractive and nonlinear terms (the r-derivative term in Eq. (2)) are performed.

The original NPE diffraction step is thus represented by

$$\frac{DR}{Dt} = -\frac{c_0}{2}\frac{R}{r} - \frac{c_0}{2}\int_{r_f}^{r}\frac{\partial^2 R}{\partial z^2}dr. \tag{3}$$

The high-angle NPE2 diffraction step (retaining terms dropped from Eq. (3)) is

$$\frac{DR}{Dt} = \int\left(-\frac{c_0}{2}\left(\frac{\partial^2 R}{\partial z^2} + \frac{R}{4r^2}\right) + \frac{1}{2c_0}\frac{D^2 R}{Dt^2}\right)dr. \tag{4}$$

For the sake of finite differences, we represent timestep number n by superscript n, and take

$$\frac{DR}{Dt} \equiv (2\delta t)^{-1}\,(R^{n+1} - R^{n-1}) \tag{5}$$

$$\frac{D^2 R}{Dt^2} \equiv \delta t^{-2}\,(R^{n+1} - 2R^{n} + R^{n-1}). \tag{6}$$

At timestep n, both R^n and R^{n-1} are known. The NPE2 diffraction step uses an implicit Crank-Nicholson representation for the range integral in Eq. (4), with the integrand averaged between steps $n+1$ and $n-1$. All terms involving time level $n+1$ are then taken to the lhs of the finite difference equation. Initial conditions in space are known at r_f, ahead of the first arrival where R is assumed zero. A trapezoidal rule for the r integral is used, and the solution is integrated backward in space at time level $n+1$ from r_f to all desired values of r.

NUMERICAL RESULTS

Benchmark calculations have been performed on the revised NPE2 diffraction step. An analytic solution has been obtained [1] for an initially spherical wave in an idealized waveguide with boundary conditions $R = 0$ at $z = 0$ (pressure release surface) and $\partial_z R = 0$ at $z = -D$ (perfectly reflecting bottom). This does not represent the ocean bottom properly, but does allow a solution by using the image method. The calculation grid was 201 by 101 points in the r and z directions, respectively. The grid spacing was $\delta r = 0.7$ m, $\delta z = 3.5$ m, and timestep $c_0\delta t = 1.5$ m.

The five curves in Fig. 1 are labeled by the maximum propagation angle in the initial condition. Initial conditions are defined at different ranges from the spherical wave source, and the error is zero at the initial range. As each benchmark propagates outward in range, the error first grows due to truncation errors and approximations made in deriving the NPE. The error reaches a peak and then falls. Why should this happen? As the NPE grid moves farther from the spherical wave source, the curvature of the wave decreases, i.e., the propagation angle decreases. The high-angle components

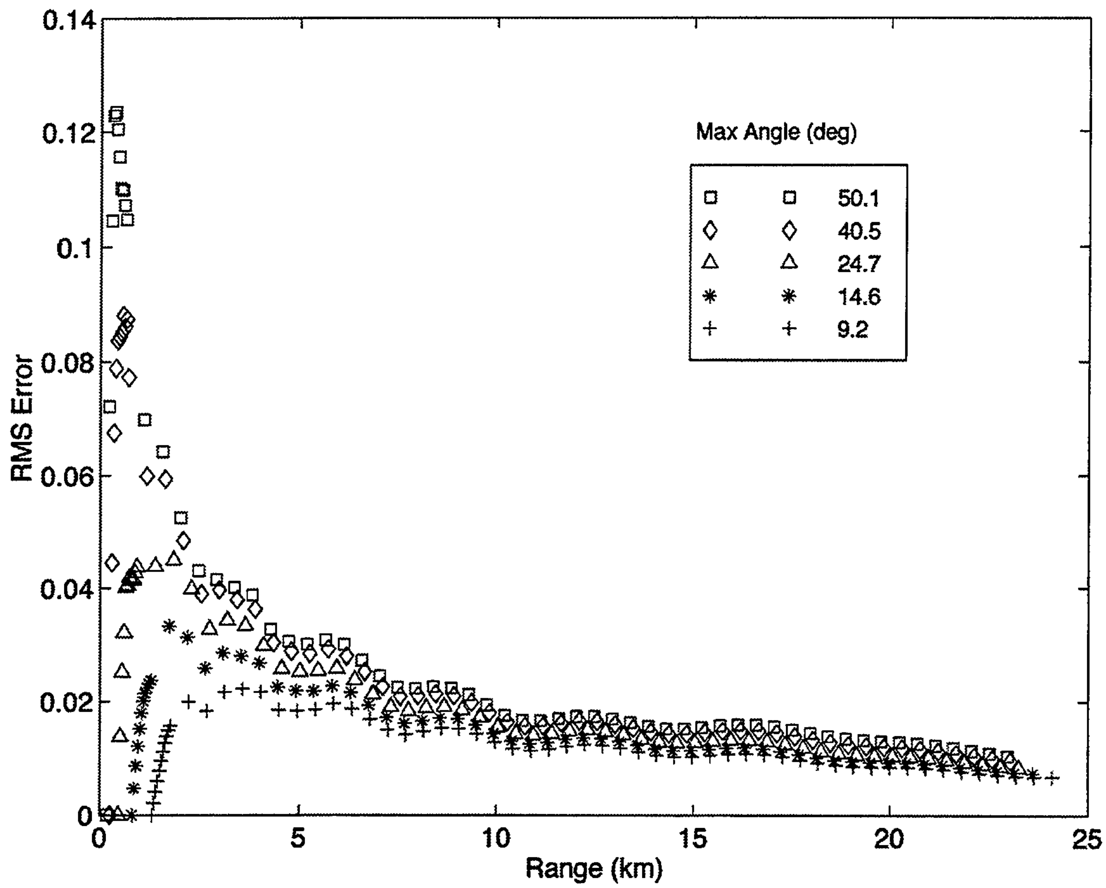

Fig. 1 — Dimensionless root-mean-square error in five different NPE2 benchmark test problems as a function of the radial distance from the center of the initially spherical wave to the left side of the moving NPE2 grid

present initially have group speeds proportional to the cosine of their propagation angle relative to the horizontal. As a result, they fall behind, being unable to keep up with the speed c_0 of the moving grid. Figure 1 shows that even for the highest propagation angle considered (50°), the maximum error peaks at approximately 1% and falls back to about 0.5%.

Figure 2 shows the result of the same set of calculations performed on the original NPE, which is first order in time. In the near field of the source at a range of 0 km, the rms errors of the highest propagation angle exceed 10%, a result of having neglected the second time derivative in the moving frame as discussed in the theory section. In the far field, however, the rms errors are approximately 1%, which is comparable to the far-field accuracy of the second order NPE2 model. Again, this is because the high-angle components of the wave have fallen behind the frame speed c_0, leaving only those low-angle components for which the first-order NPE is accurate.

SUMMARY

The results indicate that the NPE model can indeed be improved in accuracy for high propagation angles near a source. Future efforts should address similar considerations for the nonlinear terms.

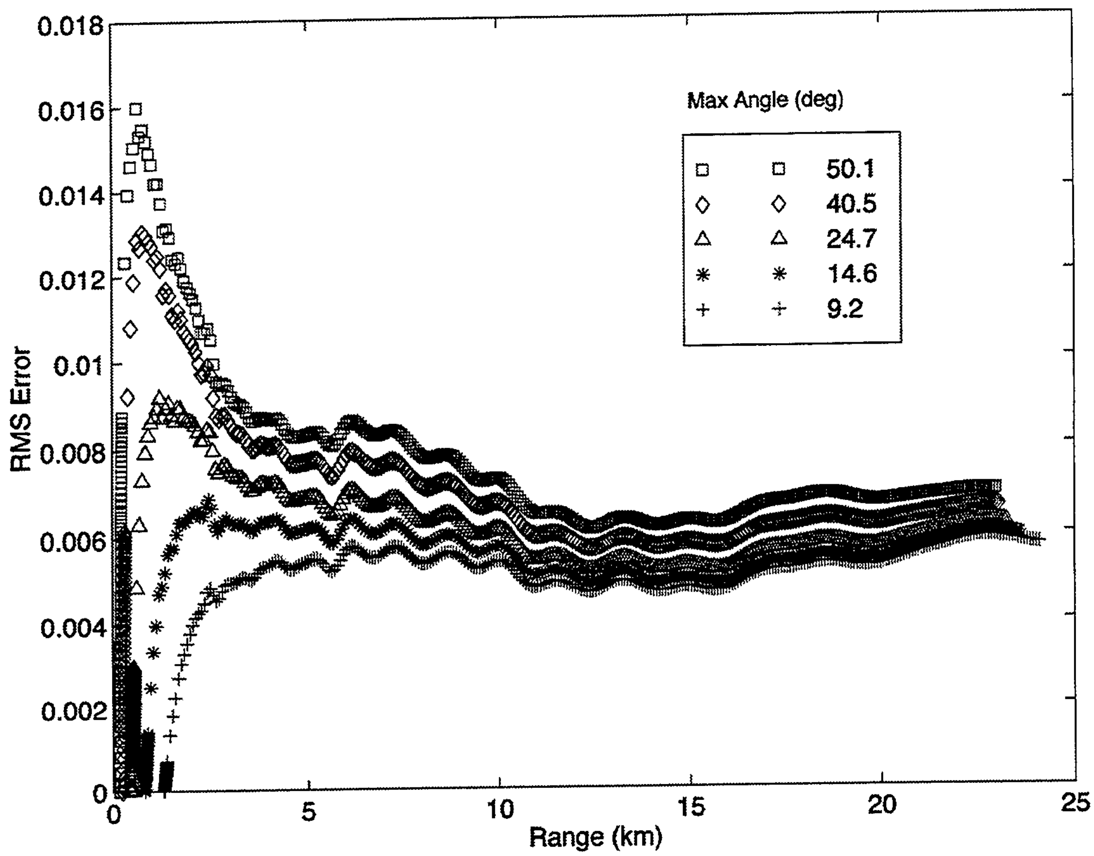

Fig. 2 — Dimensionless root-mean-square error in five different original NPE benchmark test problems as a function of the radial distance from the center of the initially spherical wave to the left side of the moving NPE2 grid

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